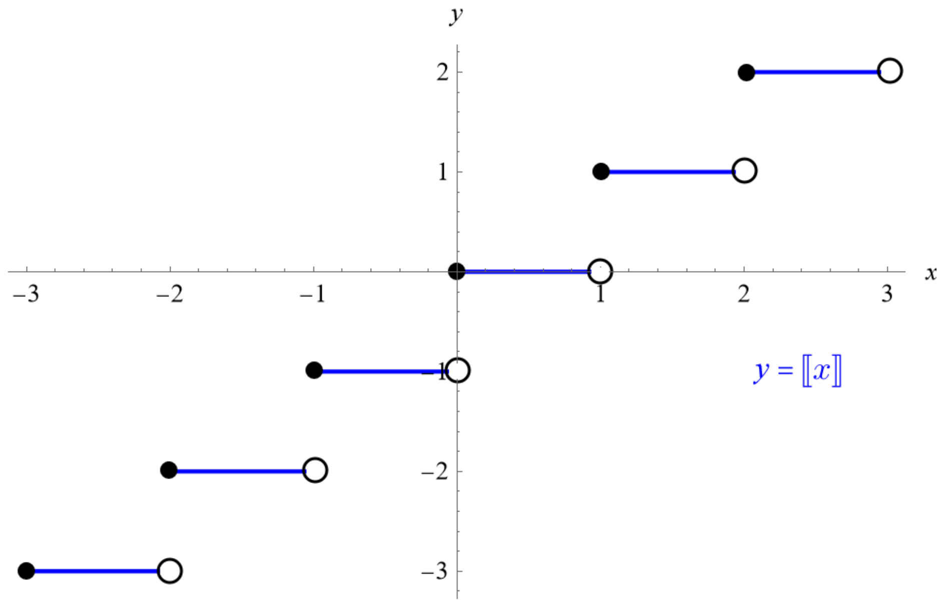
Problem 5

Evaluate the following limits, if they exist, where $[\![x]\!]$ denotes the greatest integer function.

(a) $\displaystyle\lim_{x\to 0} \frac{[\![x]\!]}{x}$ (b) $\displaystyle\lim_{x\to 0} x\,[\![1/x]\!]$

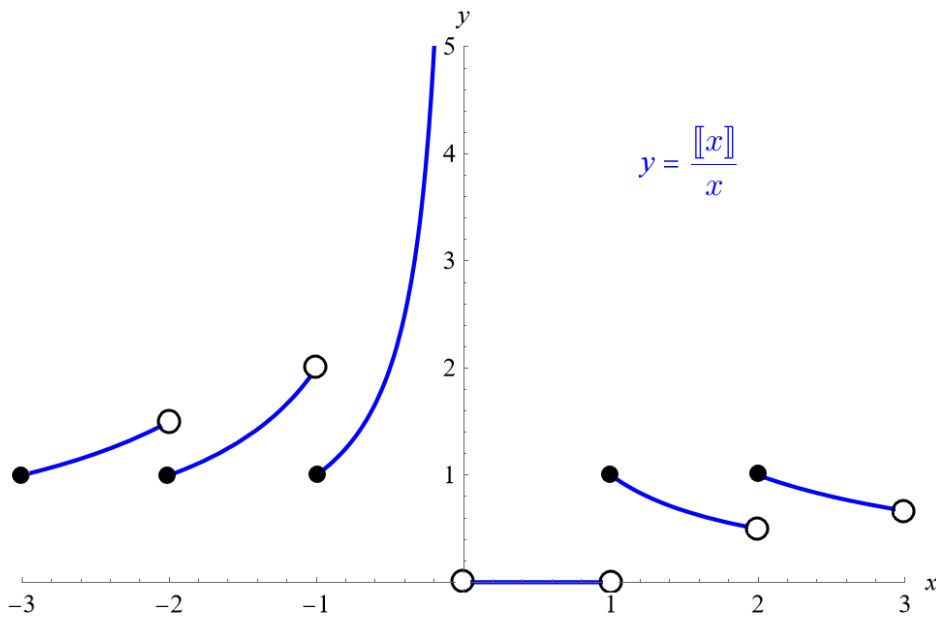
Solution

Below is the graph of the greatest integer function for reference.



Part (a)

Graph the function $\frac{[\![x]\!]}{x}$ versus x.



Observe that the left-hand and right-hand limits are not equal.

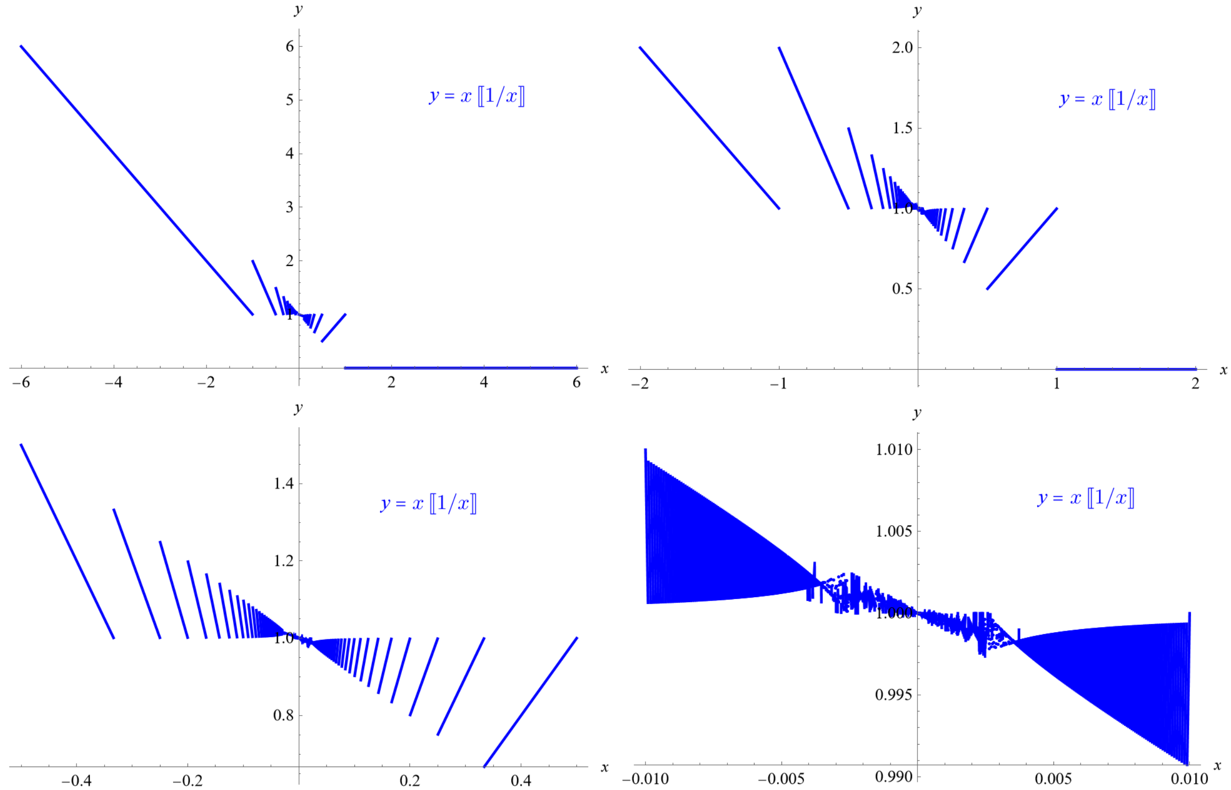
$$\lim_{x\to 0^-} \frac{[\![x]\!]}{x} = +\infty \qquad \lim_{x\to 0^+} \frac{[\![x]\!]}{x} = 0$$

Therefore,

$$\lim_{x\to 0} \frac{[\![x]\!]}{x} \quad \text{does not exist.}$$

Part (b)

The function $x\,[\![1/x]\!]$ is graphed below over several intervals of x in order to illustrate the function's behavior towards the origin.



Observe that the left-hand and right-hand limits are equal.

$$\lim_{x\to 0^-} x\,[\![1/x]\!] = 1 \qquad \lim_{x\to 0^+} x\,[\![1/x]\!] = 1$$

Therefore,

$$\lim_{x\to 0} x\,[\![1/x]\!] = 1.$$